



# The component procurement problem for the loss-averse manufacturer with spot purchase

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## ABSTRACT

This paper studies the purchasing behaviour of a loss-averse engineer-to-order manufacturer, who purchases a key component for his final product from a supplier under a single-wholesale-price contract with spot purchase opportunities, where both the product demand and the component spot price are uncertain. Through newsvendor type of models, we analyze several key issues, including the effects of the manufacturer's loss aversion, and the effects of demand and spot price uncertainties on the manufacturer's decision behaviour. We find that the purchasing behaviour of the loss-averse manufacturer differs from those of the risk-neutral and risk-averse ones. Specifically, we identify some sufficient conditions under which the loss-averse manufacturer may purchase a larger order quantity in advance when demand becomes more uncertain or when the price becomes more uncertain. We also discuss the two-wholesale-price contract and show that fixing the emergency supply price may lead to a smaller order quantity.

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## 1. Introduction

Many products that engineer-to-order (ETO) firms provide are expensive and customized capital goods, such as production equipments, commercial aircrafts, medical devices, large-scale communication systems and defence systems. The underlying technologies and engineering designs change from one order to another, so the associated key components are custom-designed with high probabilities of becoming obsolete and long lead times. Hsu et al. (2006) report that Chinese ETO firms have to import many critical components from overseas, such as integrated circuit (IC) chips, because there is little IC design and manufacturing capability within China, and the lead times for Chinese ETO firms to procure and import such custom-made components could be as long as two to three months. The assembly time for a customized product is normally much shorter than the lead time for procuring the critical components, so for simplicity in exposition, we assume that the assembly time is negligible in this paper. Upon receiving a customer order, an ETO firm will typically process the order in the following manner: At the beginning, it has to design the product for the customer. Before the final agreement on the product order is reached, there are many opportunities for revisions. However, even when a firm order is

confirmed, the actual demand for the key component used in the product may be different from what was expected because of unreliability of the final product assembly process. However, the ETO firm can estimate the component demand distribution based on past experience at the stage when the initial product design is proposed to the customer.

Contracts play a significant role in decentralized supply chains. Since the customized product made by the ETO firm cannot be partially satisfied, the firm must purchase extra units from the key component supplier via an emergency order when the initial component order quantity does not meet the product's actual demand. Based on the characteristics of the emergency purchase price, the supply contract arrangements between the key component supplier and the ETO firm can be classified into two kinds, namely the *two-wholesale-price contract* (Cachon, 2004; Dong and Zhu, 2007) and the *single-wholesale-price contract with spot purchase*. Under both arrangements, the ETO firm purchases an initial quantity of the component at a fixed contract price based on the initial component demand information. The difference between the two kinds of supply contract is that the emergency purchase price is also fixed under the former contract, but it is not fixed under the latter contract. Under the latter contract, the ETO firm may procure extra units of the component at the spot price, which is decided by the spot market and is unknown when the firm makes the initial procurement, so it is also called the "spot" price (Deng and Yano, 2002). Due to fluctuations in currency exchange rates, and shortages or excessive inventories at the

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supplier, the emergency purchase price is a random variable, whose distribution can be estimated by some methods such as the geometric Brownian motion (Gurnani and Tang, 1999; Li and Kouvelis, 1999). Under the single-wholesale-price contract with spot purchase, the ETO firm faces the issue of how best to purchase the component in such an uncertain environment, and needs to understand the effects of demand uncertainty, spot price uncertainty, and its own risk preference on its purchasing behaviour.

The perishable product purchase problem has been commonly studied via the newsvendor (or newsboy) model (Khouja, 1999). It has been shown that the “newsvendor” in the classical newsvendor model must bear all the demand risk. So a variety of supply contracts, such as the buy back arrangement (Pasternack, 1985), the quantity flexibility contract (Barnes-Schuster et al., 2002; Eppen and Iyer, 1997), the sales rebate contract (Krishnan et al., 2004; Taylor, 2002), the advance-purchase discount Contract (Cachon, 2004; Dong and Zhu, 2007), etc., have been proposed to share demand risk in supply chains. These contracts are usually studied under the assumption that there is no supply uncertainty. There are only a few studies that incorporate spot purchases into supply contracts in the context of perishable supply chain management (Gurnani and Tang, 1999; Deng and Yano, 2002). These studies are all based on the expected cost criterion, i.e., the ETO firm is assumed to be risk-neutral. On the other hand, there is a considerable body of research on component purchase and inventory management with explicit considerations of the risk factors, e.g., Lau (1980), Ritchken and Tapiero (1986), Bouakiz and Sobel (1992), Eeckhoudt et al. (1995), Agrawal and Seshadri (2000), Chen and Federgruen (2000), Gaur and Seshadri (2003), and Chen et al. (2007), among others. All these studies are based on the rational principle assumption.

However, there is growing evidence that many human decisions defy the rationality assumption. Through experiments, Kahneman and Tversky (1979) find that most decision makers have different perceived values for equal gain and loss, e.g., they suffer greater pain from a loss than they derive pleasure from an equal magnitude of gain. Such findings can be accounted for by Prospect Theory. Some researchers have studied the newsvendor problem based on Prospect Theory. In the context of the newsvendor problem, Schweitzer and Cachon (2000) conduct two laboratory experiments to investigate the decision bias of human decision makers. They show that the newsvendor tends to under-order for a high-profit product and over-order for a low-profit product. They argue that this over/under-ordering pattern cannot be explained by risk-averse and risk-seeking preferences. They suggest that one explanation for the experimental results is the anchoring and insufficient adjustment. Two new behaviour studies (see Benzion et al., 2008; Bolton and Katok, 2008) further verify Schweitzer and Cachon's results. Note that Schweitzer and Cachon (2000) show that a loss-averse newsvendor (with no shortage cost) will order strictly less than the risk-neutral newsvendor, which implies that loss aversion cannot explain the over-order pattern in their experiments. Wang and Webster (2009) extend Schweitzer and Cachon's model by considering the shortage cost. They show that the newsvendor may order more than the profit-maximization (risk-neutral) order. But they do not compare the loss-averse solution with the corresponding risk-averse solution. They further extend their work to the supply chain and competitive settings (see Wang and Webster, 2007; Wang 2010a,b). While the above research studies human decision behaviours, Fiegenbaum and Thomas (1988) empirically show that Prospect Theory can be applied to account for corporate decision behavior at both firm and industry levels. For example, Airline Financial News reported on 18 February 2002 that “the airline industry is experiencing an unprecedented level of stored

aircrafts due to a cyclical downturn and the Sept. 11 attacks, and rational economic behaviour is not prevailing”. This would strongly imply that the Prospect Theory developed by Kahneman and Tversky (1979) can be applied “to better understand the key drivers behind deciding whether to scrap or not”.

Following this stream of research, we study the purchasing behaviour of a loss-averse ETO manufacturer under a single-wholesale-price contract with spot purchase. Our problem setting is similar to Eeckhoudt et al. (1995), where they address a newsvendor problem with an emergency supply opportunity to meet all the shortage. They focus on analyzing the impact of risk aversion on the optimal ordering decision. They show that a risk-averse seller always orders less than a risk-neutral seller does. We generalize their problem by allowing the emergency supply price to be random and investigate the effect of loss aversion on the optimal ordering decision. Similar to Wang and Webster (2009), we identify the conditions under which a loss-averse manufacturer either always orders more or always orders less than the risk-neutral manufacturer does. To investigate the difference between the effects of loss aversion and risk aversion, we conduct numerical experiments. The numerical results show that when the emergency supply price is greater than the selling price, risk aversion does not provide a definitive prediction of whether the risk-averse solution is greater or less than the risk-neutral one. This implies that the loss-averse seller may have a different behaviour from the risk-averse seller. It is noted that although our findings on the effects of loss aversion on the ordering decision are similar to those of Wang and Webster (2009), they do not compare their solution to the corresponding risk-averse newsvendor solution as we do. Instead, they focus on investigating the role of shortage cost in decision bias, which generalizes Schweitzer and Cachon's (2000) analysis, whereas we concentrate on the difference between loss aversion and risk aversion. We also investigate how changing demand and supply price risks affect the optimal ordering decision of the manufacturer, and the effect of the two-wholesale-price contract on the ordering decision. Our results shed new light on behavioral analysis in an uncertain operational environment.

The remainder of this paper is organized as follows. In Section 2 we introduce and formulate the problem. In Section 3 we provide structural analysis of the optimal policy. We also address some important managerial issues, such as how the manufacturer's purchasing behavior is affected by his loss-aversion attitude, demand uncertainty, and emergency supply price uncertainty. In Section 4 we extend the model with Bernoulli spot price distribution to the case with a general spot price distribution. In Section 5 we conclude the paper and suggest topics for future research. We present the proofs in the Appendix.

## 2. Problem description and model formulation

We consider an ETO manufacturer who purchases a custom-component from a key supplier after receiving a potential customer order. Due to the long lead time, the component supplier will offer the ETO firm a single-wholesale-price contract with spot purchase. Under this contract, the ETO firm must make an advanced procurement at the fixed contract price  $w$  at time 0, when the potential customer order arrives. Because of uncertainties in product design and manufacturing processes, the component demand quantity  $D$ , which is used in the manufacturing process at a (known) future time  $t_0$ , is not known exactly at time 0, but its distribution function  $F(\xi)$  and the corresponding density function  $f(\xi)$  can be estimated. In the ETO setting, especially the heavy equipment industry, the final product cannot be delivered even if there is a shortage of only one unit of the

component. The shortage quantity from the initial procurement must be replenished from the supplier at the spot price  $p$  through an emergency supply order. As observed by Gurnani and Tang (1999), and Li and Kouvelis (1999), there are many factors such as fluctuations in currency exchange rates, shortages, and excessive inventories of the component at the supplier, etc., that will make the spot price of the emergency order  $p$  uncertain, which can go up to  $p_h > w$  with a probability of  $\alpha$ , i.e.,  $P(p > w) = \alpha$ ; or down to  $p_l \leq w$  with a probability of  $1 - \alpha$ , i.e.,  $P(p \leq w) = 1 - \alpha$ . We assume that the component supplier has unlimited capacity and can meet the manufacturer's demand,  $F(\xi) = 0$  for all  $\xi \leq 0$ , and  $f(\xi) > 0$  for all  $\xi > 0$ . Let  $\bar{F}(\xi) = 1 - F(\xi)$  and  $\bar{p} = \alpha p_h + (1 - \alpha)p_l$ .

Let  $Q$  be the component order quantity at time 0. Then the manufacturer may need to purchase  $(D - Q)^+$  extra units of the component via the emergency supply mode, and may have a surplus of  $(Q - D)^+$  units after delivery of the customer order. Let  $r$  and  $v$  be the unit revenue and unit salvage value of the component, respectively. For the order specific component, the value of  $v$  is close to zero. These parameters represent three key economic characteristics of the component. It is reasonable to assume that  $r > w \geq v$  and  $p_l > v$ ; otherwise, it might not be profitable to produce at all or it might be profitable to procure an unlimited number of the component in order to salvage them. Based on the above assumptions, the manufacturer's profit function can be expressed as

$$\pi(Q; D, p) = \begin{cases} \pi_1(Q; D, p) = (r - v)D - (w - v)Q & \text{if } D < Q, \\ \pi_2(Q; D, p) = (r - p)D - (w - p)Q & \text{if } D \geq Q. \end{cases} \quad (1)$$

Let  $d_1(Q)$  and  $d_2(Q; p)$  be the break-even quantities of the realized demand, i.e.,

$$d_1(Q) = \frac{w - v}{r - v} Q, \quad d_2(Q; p) = \begin{cases} \frac{p - w}{p - r} Q & \text{if } p > r, \\ \infty & \text{if } p \leq r. \end{cases}$$

The break-even quantities imply that if the realized demand quantity  $D = \xi$  is either too low, i.e.,  $\xi < d_1(Q)$ , or too high, i.e.,  $\xi > d_2(Q; p_h)$ , then the manufacturer's profit will be negative, i.e., incurring a loss. Otherwise, if the demand is between  $d_1(Q)$  and  $d_2(Q; p_h)$ , then the profit will be positive, i.e., making a gain.

We assume that the manufacturer has the following utility function of constant loss aversion (Kahneman and Tversky, 1979)

$$u(x) = \begin{cases} x & \text{if } x \geq 0, \\ \lambda x & \text{if } x < 0, \end{cases} \quad (2)$$

where  $\lambda > 1$  is defined as the manufacturer's loss-aversion coefficient. A larger value of  $\lambda$  indicates a higher level of loss aversion. Note that the above utility function is concave and so is the risk-averse utility function. The critical difference between a loss-averse utility function and a risk-averse utility is that the loss-averse utility function explicitly addresses the reference-dependence effect while the risk-averse utility function does not. Loss aversion has been widely employed to perform behavioural analysis in economics, finance, and operations management (see, e.g., Kahneman and Tversky, 1979; Barberis and Huang, 2001; Schweitzer and Cachon, 2000). Our research is also in this fashion.

The loss-averse ETO manufacturer's problem can be formulated as maximization of the expected loss-averse utility function. Note that  $u(x) = x - (\lambda - 1)x^-$ . The expected utility function for the loss-averse ETO manufacturer can be written as follows:

$$\Pi(Q; \lambda) = E\pi(Q; D, \bar{p}) - \Psi(Q; \lambda), \quad (3)$$

where

$$E\pi(Q; D, \bar{p}) = \int_0^Q \pi_1(Q; \xi, \bar{p}) dF(\xi) + \int_Q^\infty \pi_2(Q; \xi, \bar{p}) dF(\xi),$$

and

$$\Psi(Q; \lambda) = -(\lambda - 1) \left[ \int_0^{d_1(Q)} \pi_1(Q; \xi, \bar{p}) dF(\xi) + \alpha \int_{d_2(Q; p_h)}^\infty \pi_2(Q; \xi, p_h) dF(\xi) \right].$$

Note that  $E\pi(Q; D, \bar{p})$  is the expected profit and  $\Psi(Q; \lambda)$  is the premium paid for loss aversion. By the definitions of the break-even quantities of the realized demand, the second term on the right hand side of (3), i.e., the biased loss relative to the expected profit, is negative or zero. This means that  $\Pi(Q; \lambda) \leq E\pi(Q; D, \bar{p})$ , i.e., the expected utility of the loss-averse manufacturer is never more than its expected profit. Thus the expected utility function for the loss-averse manufacturer is the sum of its expected profit and its biased loss relative to the expected profit. Then the component purchasing problem for the loss-averse ETO manufacturer can be expressed as

$$\max_{Q \geq 0} \Pi(Q; \lambda). \quad (4)$$

### 3. Optimal policy and analysis

The component procurement behaviour of the loss-averse manufacturer can be studied through the stochastic program (4). The following proposition establishes that there exists a unique optimal solution to problem (4).

**Proposition 1.** *The expected utility function for the loss-averse manufacturer,  $\Pi(Q; \lambda, \alpha)$ , is concave for all  $Q$  in the range of the demand distribution function  $F(\xi)$ . So there is a unique optimal solution to problem (4),  $Q_\lambda^*$ , which satisfies the following first-order condition:*

$$(\bar{p} - w) - (\bar{p} - v)F(Q) - (\lambda - 1)\psi(Q) = 0, \quad (5)$$

where

$$\psi(Q) := (\partial \Psi(Q; \lambda) / \partial Q) / \lambda - 1 = (w - v)F(d_1(Q)) - \alpha(p_h - w)\bar{F}(d_2(Q; p_h)).$$

This proposition establishes the concavity of the objective function, which ensures the optimality of the solution (5). Note that  $(\bar{p} - w) - (\bar{p} - v)F(Q)$  is the marginal profit of inventory and  $(\lambda - 1)\psi(Q)$  is the premium paid for loss aversion per unit inventory. In the following, without causing any confusion, we denote  $\psi(Q)$  as the *marginal premium of loss aversion*. Thus the first-order Eq. (5) implies that, under the optimal decision, the marginal profit equals the marginal premium of loss aversion multiplied by  $(\lambda - 1)$ . In particular, when the manufacturer is risk-neutral, i.e.,  $\lambda = 1$ , (5) yields the result of Gurnani and Tang (1999).

#### 3.1. Effects of loss aversion

We now discuss the impacts of the manufacturer's loss-aversion on the purchasing decision. We define  $\underline{\gamma} = (w - v) / \alpha(p_h - w)$  as the manufacturer's minimum cost ratio of the unit absolute overage cost to the expected unit absolute underage cost, and  $\gamma_\lambda = \bar{F}(d_2(Q_{\lambda, \alpha}^*; p_h)) / F(d_1(Q_{\lambda, \alpha}^*))$  as the ratio of the probability of having the absolute underage loss to the probability of having the absolute overage loss. Similar to Wang and Webster (2009), we obtain the following proposition that demonstrates the effect of loss aversion on the optimal ordering quantity.

**Proposition 2.** *If  $\gamma_1 > \underline{\gamma}$ , then  $Q_{\lambda_1}^* \geq Q_{1, \alpha}^*$  and  $Q_{\lambda_1}^*$  is increasing in  $\lambda$ ; if  $\gamma_1 < \underline{\gamma}$ , then  $Q_{\lambda_1}^* \leq Q_{1, \alpha}^*$  and  $Q_{\lambda_1}^*$  is decreasing in  $\lambda$ ; and if  $\gamma_1 = \underline{\gamma}$ , then  $Q_{\lambda_1}^* = Q_{1, \alpha}^*$  and  $Q_{\lambda_1}^*$  is independent of  $\lambda$ .*

This proposition shows that the loss-averse manufacturer will order more if the loss ratio of the risk-neutral one is higher than the minimum cost ratio and order less if the loss ratio of the risk-neutral one is lower than the minimum cost ratio.

The condition  $\gamma_1 > \underline{\gamma}$  ( $\gamma_1 < \underline{\gamma}$ ) implies that  $\psi(Q_1^*) < 0$  ( $\psi(Q_1^*) > 0$ ), i.e., the marginal premium of loss aversion at the risk-neutral optimal quantity is negative (positive). Thus the above proposition can also be interpreted as follows: If the marginal premium of loss aversion at the risk-neutral optimal quantity is negative, then the loss-averse newsvendor will order more than the risk neutral one and the more loss-averse the decision maker is, the more he will order; if the marginal premium of loss-aversion at the risk-neutral optimal quantity is positive, then the loss-averse newsvendor will order less than the risk neutral one and the less loss-averse the decision maker is, the less he will order; and if the marginal premium of loss aversion at the risk-neutral optimal quantity is zero, then loss aversion does not affect the ordering decision.

The following example demonstrates the effects of loss aversion on the optimal ordering decisions.

**Example 1.** Set  $r = 1, \alpha = p_l = w = 0.5, v = 0, F(\xi) = \frac{1}{50}e^{-\xi/50}$ , and  $p_h \in \{1.6, 2.0\}$ . Change the value of the parameter  $\lambda$  to generate the optimal decision curves for  $p_h = 1.6, 2.0$ , respectively (see Fig. 1). We can observe that the optimal ordering quantity increases in  $\lambda$  when  $p_h = 1.6$  but decreases in  $\lambda$  when  $p_h = 2.0$ . Note that  $\gamma_1 = 0.83 < \underline{\gamma} = 0.91$  when  $p_h = 1.6$ , and  $\gamma_1 = 0.69 > \underline{\gamma} = 0.67$  when  $p_h = 2.0$ . This confirms the prediction of Proposition 2.

Note that if  $p_h \leq r$  then  $d_2(Q; p) = \infty$  and  $\gamma_1 = 0 < \underline{\gamma}$ . Applying Proposition 2, we have the following corollary.

**Corollary 3.** If  $p_h \leq r$ , then  $Q_\lambda^*$  is decreasing in  $\lambda$  and  $Q_\lambda^* \leq Q_{1,\alpha}^*$ .

This corollary states that when the possible highest emergency procurement price is lower than the marginal revenue, the loss-averse manufacturer will always order less than the risk-neutral manufacturer. In addition, the more loss-averse the manufacturer is, the smaller the initial order quantity he will place. Naturally, more order quantities will be deferred to the second opportunity.

A natural question arises here: Does a loss-averse decision maker behave differently from a risk-averse decision maker? Note that [Eeckhoudt et al. \(1995\)](#) show that the risk-averse decision maker always orders less than the risk-neutral one, assuming that the emergency supply cost is lower than the selling price. Corollary 3 shows that loss aversion has a similar effect on the ordering decision when  $p_h \leq r$ . But it is unclear whether the result holds under the risk-averse framework when the emergency supply cost is higher than the selling price. Neither [Schweitzer and Cachon \(2000\)](#) nor [Wang and Webster \(2009\)](#) address this question. To gain some insights, we use numerical experiments to compare the patterns of risk-averse solutions and loss-averse solutions.

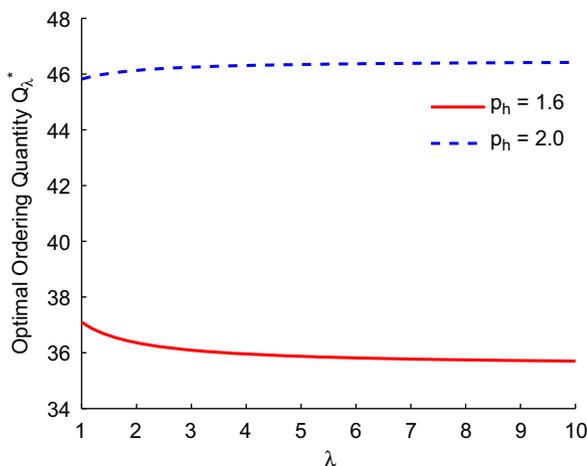


Fig. 1. Effects of loss aversion.

**Example 2.** Set  $r = 1, \alpha = 1, w = 0.5, v = 0, p_l = 0.5, p_h = 1.6$ , and  $F(\xi) = \frac{1}{50}e^{-\xi/50}$ . Note that  $p_h > r$ . For comparison purposes, we consider a loss-averse manufacturer with a degree of loss aversion being characterized by  $\lambda$  and a risk-averse manufacturer with a utility function  $U(x) = -e^{-\gamma x}$ , where  $\gamma$  represents the degree of risk aversion. Change the values of the parameters  $\gamma$  and  $\lambda$  to generate the curves of the optimal ordering quantities (see Figs. 2 and 3).

Fig. 2 shows that when the degree of risk aversion is relatively large (small), a risk-averse decision is greater (less) than the risk-neutral decision. However, the loss-averse decision is always smaller than the risk-neutral decision, as shown in Fig. 3. Note that  $\gamma_1 = 0.27 < \underline{\gamma} = 0.45$ . By Proposition 2, we know that the loss-averse decision must be greater than the risk-neutral one and increasing in  $\lambda$ . This indicates that the loss-averse decision maker indeed has a different behavioral pattern from the risk-averse one.

### 3.2. Effects of increased demand risk

We next analyze the effects of an increase in demand risk on the optimal decisions. Changes in risk are represented by a change in the demand distribution from  $F(\xi)$  to  $G(\xi)$ . Without loss of generality, we assume that the support of  $G$  is contained in  $[0, \infty)$ . Let  $Q_{\lambda,\alpha,F}^*$  denote the optimal order quantity with demand distribution  $F(\xi)$  and  $Q_{\lambda,\alpha,G}^*$  denote the optimal order quantity with demand distribution  $G(\xi)$ . The corresponding expected utility

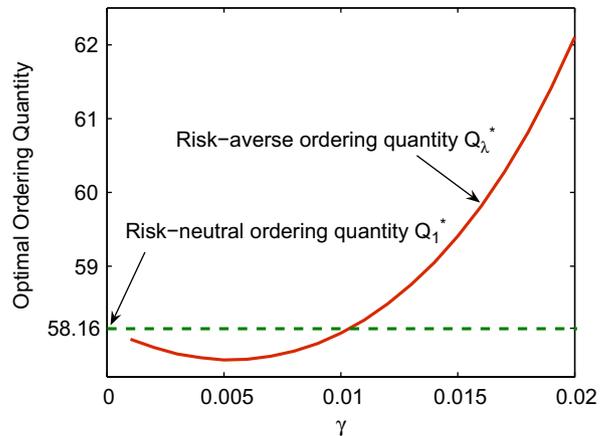


Fig. 2. Risk-averse decision.

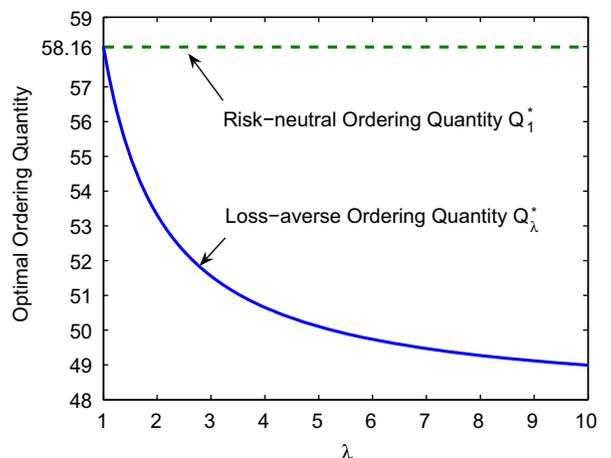


Fig. 3. Loss-averse decision.

functions are denoted by  $\Pi_F$  and  $\Pi_G$ , respectively. We say that  $F$  first-order stochastically dominates  $G$  if  $F(x) \leq G(x)$  for all  $x \in [0, \infty)$ . Under the first-order stochastic dominance condition, we find that the optimal initial order quantity decreases with increasing risk, which is stated in the following proposition:

**Proposition 4.** *If  $F$  first-order dominates  $G$  in  $[0, \infty)$ , then  $Q_{i,G}^* \leq Q_{i,F}^*$ .*

This proposition fits our intuition that a larger demand (in probabilistic sense) implies a greater ordering quantity.

Similar to [Eeckhoudt et al. \(1995\)](#), we give the following definition of mean-preserving increase in risk (MPIR):

**Definition 1** (MPIR, [Rothschild and Stiglitz, 1970](#)).  $G$  is said to be a mean-preserving increase in risk (MPIR) of  $F$  if  $G$  and  $F$  satisfy the following two conditions:

$$\int_0^t (G(\xi) - F(\xi)) d\xi \geq 0 \quad \text{for all } t \in [0, \infty) \text{ (increase in spread),} \quad (6)$$

$$\int_0^\infty (G(\xi) - F(\xi)) d\xi = 0 \text{ (mean-preservation).} \quad (7)$$

While condition (6) states the second-order stochastic dominance of  $F$  over  $G$ , condition (7) preserves the mean ([Eeckhoudt et al., 1995](#)). Based on the above definition, we have the following proposition:

**Proposition 5.** (1) *An MPIR restricted to the interval  $[0, \infty)$  reduces the optimal expected loss-averse utility, i.e.,  $\Pi_G^*(Q_{i,G}^*; \lambda) \leq \Pi_F^*(Q_{i,F}^*; \lambda)$ .* (2) *If  $(\bar{p} - v)(G(Q_{i,F}^*) - F(Q_{i,F}^*)) + (\lambda - 1)[\alpha(p_h - w)(G(d_2(Q_{i,F}^*; p_h)) - F(d_2(Q_{i,F}^*; p_h))) + (w - v)(G(d_1(Q_{i,F}^*)) - F(d_1(Q_{i,F}^*)))] \geq 0$ ,* (8)

*then an MPIR restricted to the interval  $[0, \infty)$  decreases the optimal order quantity, i.e.,  $Q_{i,F}^* \geq Q_{i,G}^*$ ; otherwise  $Q_{i,F}^* < Q_{i,G}^*$ .*

Proposition 4 states that, with the MPIR restriction, (1) the optimal order quantity for the loss-averse manufacturer with demand distribution  $G$  can be less or more than that with demand distribution  $F$  when  $F$  stochastically dominates  $G$ , and (2) the maximum expected utility increases from  $G$  to  $F$ , which means that the loss-averse manufacturer prefers demand distribution  $F$  to demand distribution  $G$ .

To obtain unambiguous results, another concept is needed.

**Definition 2.**  $G$  is an increase in the single spread across  $\beta$  on  $F$  if  $G$  and  $F$  satisfy the following condition:

$$[G(y) - F(y)][\beta - y] \geq 0. \quad (9)$$

Inequality (9) is also called the *single-crossing condition*. It implies that the distribution curves spread across only once and  $F$  dominates  $G$  conditionally on  $y < \beta$  and  $G$  dominates  $F$  conditionally on  $y > \beta$ . If the simple spread also preserves the mean, then (9) and (6) are implied, which implies that it is a special case of MPIR ([Rothschild and Stiglitz, 1970](#)). For example, if  $F$  and  $G$  follow the normal distribution with standard deviations  $\sigma_F < \sigma_G$  and a common mean  $\mu > 0$ , then it is easy to verify that (9) holds when  $\beta = \mu$ , which implies that a change from  $F$  to  $G$  is a simple spread across  $\mu$ .

The following proposition shows the impact of increased demand risk on the optimal decisions under loss aversion.

**Proposition 6.** *Suppose  $G$  is an increase in the single spread across  $\beta$  on  $F$ .*

(a) *If  $d_2(Q_{i,F}^*) \leq \beta$  and  $\gamma_{1F} \leq \underline{\gamma}$ , where  $\gamma_{1F} = \bar{F}(d_2(Q_{i,F}^*; p_h)) / F(d_1(Q_{i,F}^*))$ , then  $Q_{i,F}^* \geq Q_{i,G}^*$ .*

(b) *If  $p_h \leq r$  and  $Q_{i,F}^* \leq \beta$ , then  $Q_{i,F}^* \geq Q_{i,G}^*$ .*

(c) *If  $d_1(Q_{i,F}^*) \geq \beta$  and  $\gamma_{1F} \geq \underline{\gamma}$ , then  $Q_{i,F}^* \leq Q_{i,G}^*$ .*

This proposition provides sufficient conditions under which an increase in the demand risk in the sense of condition (9) decreases or increases the ordering quantity. Under the single-crossing condition (9), [Eeckhoudt et al. \(1995\)](#) show that a risk-averse decision maker orders less than the risk neutral decision maker does. Our results show that the model under loss aversion framework can have richer predictions on the effects of increased demand risk. Moreover, these conditions are much sharper than those provided by [Eeckhoudt et al. \(1995\)](#) in that our conditions are only defined on the risk-neutral solution while applying to any degree of loss aversion, whereas theirs are restricted to a given risk-averse utility function.

### 3.3. Effects of increased price risk

We now analyze the effects of increased emergency supply price risk on the optimal decision. An increase in supply price risk can be represented by a change from the emergency supply price distribution  $F_p(y)$  to distribution  $G_p(y)$ , which can be denoted as follows:

$$F_p(y) = \begin{cases} 0 & \text{if } y < p_l, \\ 1 - \alpha & \text{if } p_l \leq y < p_h, \\ 1 & \text{if } y \geq p_h \end{cases}$$

and

$$G_p(y) = \begin{cases} 0 & \text{if } y < p'_l, \\ 1 - \alpha' & \text{if } p'_l \leq y < p'_h, \\ 1 & \text{if } y \geq p'_h. \end{cases}$$

Then a change from  $F_p$  to  $G_p$  is a simple spread across  $p_h$  if and only if  $p_l \geq p'_l, \alpha \geq \alpha'$ , and  $p_h \leq p'_h$ .

**Proposition 7.** *Suppose  $G_p$  is an increase in the single spread across  $p_h$  on  $F_p$  and the single spread preserves the mean. Then,  $Q_{i,F_p}^* \leq Q_{i,G_p}^*$ .*

Proposition 7 shows that increased price risk (in the sense of a single spread with mean preservation) leads to an increase in ordering quantity. The rationale is that as the emergency supply price risk increases, a loss-averse manufacturer will order more to reduce the potential emergency supply order.

### 3.4. Two-wholesale-price contracts versus single-wholesale-price contracts

By now we have studied the procurement problem for the loss-averse manufacturer under the single-wholesale-price contract. In practice, to reduce the price risk, the purchaser may also want to lock in the emergency supply price with the same supplier by signing a contract containing an initial order price  $w$  and an emergency order price  $\bar{p}$ , which is called the two-wholesale-price contract ([Cachon, 2004; Dong and Zhu, 2007](#)), denoted as  $(w, \bar{p})$ . By offering the two-wholesale-price contract, the corresponding supplier shares the risk with the manufacturer due to the obligation to meet all the demand shortage. Let  $\tilde{Q}_\lambda^*$  be the optimal initial order quantity for the loss-averse manufacturer under the two-part pricing contract  $(w, \bar{p})$ . We have the following result.

**Proposition 8.** *If*

$$\bar{p}\bar{F}(Q_\lambda^*) + (\lambda - 1)(\bar{p} - w)\bar{F}(d_2(Q_\lambda^*; \bar{p})) \leq \bar{p}\bar{F}(Q_\lambda^*) + \alpha(\lambda - 1)(p_h - w)\bar{F}(d_2(Q_\lambda^*; p_h)),$$

*then  $\tilde{Q}_\lambda^* \leq Q_\lambda^*$ . Otherwise,  $\tilde{Q}_\lambda^* > Q_\lambda^*$ . In particular, if  $\bar{p} \leq \min(r, \bar{p})$ , then  $\tilde{Q}_\lambda^* \leq Q_\lambda^*$ .*

This proposition shows that under the two-wholesale-price contract the optimal ordering quantity may be greater or less than the optimal ordering quantity under the single-wholesale-price contract. When the emergency supply price  $\bar{p}$  is less than the selling price  $r$  and the average spot price  $\bar{p}$ , the two-part pricing contract yields a smaller ordering quantity. From the supplier's point of view, this seems to be counterintuitive: Sharing the demand risk with the buyer by offering a two-part pricing contract in fact leads to a smaller ordering quantity. Recall that Proposition 7 provides the insight that increased price risk increases ordering quantity. Thus, the reduction in the price risk due to the lower fixed emergency supply price induces the loss-averse manufacturer to reduce the ordering quantity. This provides a behavioral insight for supply chain management.

**4. Extension to general spot price distribution**

A restriction of the preceding analysis is the assumption that  $p \in \{p_h, p_l\}$  for the sake of tractability. In this section we relax this restriction by assuming that  $p$  is distributed in  $[v, \infty)$ , where  $\bar{p} = E[p]$ , with the distribution function  $G$ . Assume that the demand and price are independent. Then the expected utility function  $\Pi$  can be expressed as

$$\Pi(Q; \lambda) = E\pi(Q; D, p) - \Psi(Q; \lambda),$$

where

$$\Psi(Q; \lambda) = -(\lambda - 1) \left[ \int_0^{d_1(Q)} \pi_1(Q; \xi, \bar{p}) dF(\xi) + \int_r^\infty \int_{d_2(Q; p)}^\infty \pi_2(Q; \xi, p) dF(\xi) dG(p) \right].$$

Taking the first derivative of  $\Pi$  with respect to  $Q$  yields

$$\frac{\partial \Pi(Q; \lambda)}{\partial Q} = (\bar{p} - w) - (\bar{p} - v)F(Q) - (\lambda - 1)\psi(Q),$$

where  $\psi(Q) = (w - v)F(d_1(Q)) - \int_r^\infty (p - w)\bar{F}(d_2(Q; p))dG(p)$ .

Since  $d_1(Q)$  and  $d_2(Q; p)$  are increasing in  $Q$  for any given  $p > r$ , the right-hand side of the above equality is decreasing in  $Q$ , which implies that  $\Pi$  is concave in  $Q$ . The optimal quantity decision  $Q_\lambda^*$  satisfies the following first-order condition:

$$(\bar{p} - w) - (\bar{p} - v)F(Q) - (\lambda - 1)\psi(Q) = 0.$$

Similar to Proposition 2, we have the following proposition. The proof, which is similar to that of Proposition 2, is skipped.

**Proposition 9.** *If  $\psi(Q_1^*) < 0$ , then  $Q_\lambda^* \geq Q_1^*$  and  $Q_\lambda^*$  is increasing in  $\lambda$ ; if  $\psi(Q_1^*) > 0$ , then  $Q_\lambda^* \leq Q_1^*$  and  $Q_\lambda^*$  is decreasing in  $\lambda$ ; and if  $\psi(Q_1^*) = 0$ , then  $Q_\lambda^* = Q_1^*$  for all  $\lambda \geq 1$ .*

**5. Conclusions**

For a long time, purchasing decisions have been widely studied based on the expected utility theory, i.e., on the assumption of a risk-neutral or a risk-averse decision maker. Based on Prospect Theory, we study the custom-component procurement decision problem with uncertain demand and an uncertain emergency supply price. After deriving the optimal initial order quantity for a loss-averse ETO manufacturer, we discuss some key issues, such as the effects of the manufacturer's loss aversion on the optimal initial order quantity, and how demand and emergency supply price uncertainties affect the decision behavior of the loss-averse manufacturer. We find that the purchasing behavior of the loss-averse manufacturer is different from those

of the risk-neutral and risk-averse ones. The loss-averse manufacturer may order a larger quantity in advance when demand or emergency supply price becomes more uncertain. We also consider the two-wholesale-price contract and show that the loss-averse manufacturer may order less under this contract. In summary, the purchasing behavior of the loss-averse manufacturer is more complex than those of the risk-neutral and risk-averse manufacturers. This behavior reflects real-life purchasing situations. While many researchers have examined the issues of supply contracts and supply chain coordination (e.g., Anupindi, 1999), future research should study the impacts of various supply contracts on the purchasing behavior of the loss-averse manufacturer.

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**Appendix A**

*A.1. Proof of Proposition 1*

The first derivative of the expected utility function with respect to  $Q$  is

$$\frac{\partial \Pi(Q; \lambda)}{\partial Q} = (\bar{p} - w) - (\bar{p} - v)F(Q) + (\lambda - 1) \times [\alpha(p_h - w)\bar{F}(d_2(Q; p_h)) - (w - v)F(d_1(Q))] \tag{1}$$

and the second derivative of the expected utility function with respect to  $Q$  is

$$\frac{\partial^2 \Pi(Q; \lambda)}{\partial Q^2} = -(\bar{p} - v)f(Q) - (\lambda - 1) \times \left[ f(d_1(Q)) \frac{(w - v)^2}{r - v} + \alpha f(d_2(Q; p_h)) \frac{(p_h - w)^2}{p_h - r} \right] < 0. \tag{2}$$

The the objective function is concave.

*A.2. Proof of Proposition 2*

The proof is similar to Wang and Webster (2009, Theorem 2). To be self-contained, we provide the key procedures here.

If  $\gamma_1 > \gamma_2$ , then  $\psi(Q_1^*) < 0$ . For any  $\lambda > 1$ , we have  $(\partial \Pi(Q_1^*; \lambda))/\partial Q = (\bar{p} - w) - (\bar{p} - v)F(Q_1^*) - (\lambda - 1)\psi(Q_1^*) = -(\lambda - 1)\psi(Q_1^*) > 0$ , where the second equality is derived from the first-order condition  $(\bar{p} - w) - (\bar{p} - v)F(Q_1^*) = 0$ . The concavity of  $\Pi(\cdot; \lambda)$  implies that  $Q_\lambda^* \geq Q_1^*$ . Then, the concavity of  $\Pi(\cdot; 1)$  implies that  $(\bar{p} - w) - (\bar{p} - v)F(Q_\lambda^*) \leq 0$ , which in turns implies that  $\psi(Q_\lambda^*) \leq 0$  for any  $\lambda > 1$ .

For any  $1 < \lambda_1 < \lambda_2$ , we have

$$\begin{aligned} \frac{\partial \Pi(Q_{\lambda_1}^*; \lambda_2)}{\partial Q} &= (\bar{p} - w) - (\bar{p} - v)F(Q_{\lambda_1}^*) - (\lambda_2 - 1)\psi(Q_{\lambda_1}^*) \\ &\geq (\bar{p} - w) - (\bar{p} - v)F(Q_{\lambda_1}^*) - (\lambda_1 - 1)\psi(Q_{\lambda_1}^*) \\ &= \frac{\partial \Pi(Q_{\lambda_1}^*; \lambda_1)}{\partial Q} = 0, \end{aligned} \tag{3}$$

where the inequality is due to the fact that  $\psi(Q_{\lambda_1}^*) \leq 0$ . By the concavity of  $\Pi(\cdot; \lambda_2)$ , we know that  $Q_{\lambda_1}^* \leq Q_{\lambda_2}^*$ .

The other assertions can be proved in a similar way and thus we skip the remaining proofs.

A.3. Proof of Proposition 4

If  $F$  first-order stochastically dominates  $G$ , then  $F(Q) \leq G(Q)$ ,  $F(d_2(Q; p_h)) \leq G(d_2(Q; p_h))$ , and  $F(d_1(Q)) \leq G(d_1(Q))$ . It is easy to verify that

$$\frac{\partial \Pi_G(Q_{iF}^*; \lambda)}{\partial Q} \leq \frac{\partial \Pi_F(Q_{iF}^*; \lambda)}{\partial Q} = 0.$$

Then the concavity of  $\Pi_G$  implies that  $Q_{iG}^* \leq Q_{iF}^*$ .

A.4. Proof of Proposition 5

(1) Letting  $\Delta = \Pi_F(Q; \lambda) - \Pi_G(Q; \lambda)$ , we have

$$\begin{aligned} \Delta &= \int_0^\infty u(\pi(Q; \xi, \bar{p}))d(F(\xi) - G(\xi)) \\ &= \pi_1(Q; \xi, \bar{p})[F(\xi) - G(\xi)]_0^Q + (r-v) \int_0^Q [F(\xi) - G(\xi)] d\xi \\ &\quad + \pi_2(Q; \xi, \bar{p})[F(\xi) - G(\xi)]_Q^\infty - (\bar{p}-r) \int_Q^\infty [F(\xi) - G(\xi)] d\xi \\ &\quad + (\lambda-1) \left\{ \pi_1(Q; \xi, \bar{p})[F(\xi) - G(\xi)]_0^{d_1(Q)} + (r-v) \int_0^{d_1(Q)} [F(\xi) - G(\xi)] d\xi \right. \\ &\quad \left. + \alpha \pi_2(Q; \xi, p_h)[F(\xi) - G(\xi)]_{d_2(Q; p_h)}^\infty - \alpha(p_h-r) \int_{d_2(Q; p_h)}^\infty [F(\xi) - G(\xi)] d\xi \right\}. \end{aligned}$$

Since  $F(0) = G(0) = 0$ ,  $\pi_1(Q; Q, \bar{p}) = \pi_2(Q; Q, \bar{p})$ ,  $\pi_1(Q; d_1(Q), \bar{p}) = \pi_2(Q; d_2(Q; p_h), p_h) = 0$ , and  $F(\infty) = G(\infty) = 1$ , we have

$$\begin{aligned} \Delta &= (r-v) \int_0^Q [G(\xi) - F(\xi)] d\xi - (\bar{p}-r) \int_Q^\infty [G(\xi) - F(\xi)] d\xi \\ &\quad + (\lambda-1) \left[ (r-v) \int_0^{d_1} [G(\xi) - F(\xi)] d\xi - (p_h-r) \alpha \int_{d_2(Q; p_h)}^\infty [G(\xi) - F(\xi)] d\xi \right]. \end{aligned}$$

Since  $\int_0^\infty [G(\xi) - F(\xi)] d\xi = 0$ , we have

$$\begin{aligned} \int_Q^\infty [G(\xi) - F(\xi)] d\xi &= \int_0^\infty [G(\xi) - F(\xi)] d\xi - \int_0^Q [G(\xi) - F(\xi)] d\xi \\ &= - \int_0^Q [G(\xi) - F(\xi)] d\xi, \\ \int_{d_2(Q; p_h)}^\infty [G(\xi) - F(\xi)] d\xi &= \int_0^\infty [G(\xi) - F(\xi)] d\xi - \int_0^{d_2(Q; p_h)} [G(\xi) - F(\xi)] d\xi \\ &= - \int_0^{d_2(Q; p_h)} [G(\xi) - F(\xi)] d\xi. \end{aligned}$$

So

$$\begin{aligned} \Delta &= (r-v) \int_0^Q [G(\xi) - F(\xi)] d\xi + (\bar{p}-r) \int_Q^\infty [G(\xi) - F(\xi)] d\xi \\ &\quad + (\lambda-1) \left[ (r-v) \int_0^{d_1(Q)} [G(\xi) - F(\xi)] d\xi + (p_h-r) \alpha \int_0^{d_2(Q; p_h)} [G(\xi) - F(\xi)] d\xi \right] \\ &= (\bar{p}-v) \int_0^Q [G(\xi) - F(\xi)] d\xi + (\lambda-1) \left[ (r-v) \int_0^{d_1(Q)} [G(\xi) - F(\xi)] d\xi \right. \\ &\quad \left. + (p_h-r) \alpha \int_0^{d_2(Q; p_h)} [G(\xi) - F(\xi)] d\xi \right] \geq 0. \end{aligned}$$

This implies that for any  $Q$ , we have

$$\Pi_F(Q; \lambda) \geq \Pi_G(Q; \lambda).$$

Then we have

$$\max_Q \Pi_F(Q; \lambda) \geq \max_Q \Pi_G(Q; \lambda), \text{ i.e. } \Pi_F(Q_{iF}^*; \lambda) \geq \Pi_G(Q_{iG}^*; \lambda).$$

(2) From the first-order condition (5), we have

$$\begin{aligned} \frac{\partial \Pi_F(Q_{iF}^*; \lambda)}{\partial Q} - \frac{\partial \Pi_G(Q_{iF}^*; \lambda)}{\partial Q} &= (\bar{p}-v)(G(Q_{iF}^*) - F(Q_{iF}^*)) \\ &\quad + (\lambda-1)[\alpha(p_h-w)(G(d_2(Q_{iF}^*; p_h)) \\ &\quad - F(d_2(Q_{iF}^*; p_h))) + (w-v)(G(d_1(Q_{iF}^*)) \\ &\quad - F(d_1(Q_{iF}^*)))]. \end{aligned}$$

Thus, by the concavity of the expected utility, the results are obvious to hold.

A.5. Proof of Proposition 6

We only prove part (a). Parts (b) and (c) can be proved using the same logic for proving part (a) so we omit them.

Since  $d_1(Q_{1F}^*) < Q_{1F}^* < d_2(Q_{1F}^*; p_h)$ , then  $d_2(Q_{1F}^*; p_h) \leq \beta$  implies that  $d_1(Q_{1F}^*) < Q_{1F}^* < d_2(Q_{1F}^*; p_h) \leq \beta$ . Then, by the single-crossing condition, we know that  $F(d_1(Q_{1F}^*)) \leq G(d_1(Q_{1F}^*))$ ,  $F(Q_{1F}^*) \leq G(Q_{1F}^*)$ , and  $F(d_2(Q_{1F}^*; p_h)) \leq G(d_2(Q_{1F}^*; p_h))$ . It is clear that inequality (8) is satisfied when  $\lambda = 1$ . By Proposition 5, we know that  $Q_{iF}^* \geq Q_{iG}^*$ .

Since  $\gamma_{1F} \leq \gamma$ , by Proposition 2,  $Q_{iF}^* \leq Q_{1F}^*$  and  $Q_{iF}^*$  is decreasing in  $\lambda$ . Then, for any  $\lambda > 1$ ,  $Q_{iF}^* < \beta$ . Since  $d_1(Q)$  and  $d_2(Q; p_h)$  are both increasing in  $Q$ , we have  $d_1(Q_{iF}^*) \leq d_1(Q_{1F}^*) < \beta$  and  $d_2(Q_{iF}^*; p_h) \leq d_2(Q_{1F}^*; p_h) \leq \beta$ . Then, applying the single-crossing condition, we know that  $F(d_1(Q_{iF}^*)) \leq G(d_1(Q_{iF}^*))$ ,  $F(Q_{iF}^*) \leq G(Q_{iF}^*)$ , and  $F(d_2(Q_{iF}^*; p_h)) \leq G(d_2(Q_{iF}^*; p_h))$ , which implies that inequality (8) is satisfied for any  $\lambda > 1$ . By Proposition 5,  $Q_{iF}^* \geq Q_{iG}^*$  for any  $\lambda > 1$ . This proves part (a).

A.6. Proof of Proposition 7

Note that for any  $Q > 0$ ,  $d_2(Q; p_h) = (p_h - w / p_h - r)Q = Q + (r - w / p_h - r)Q$  is decreasing in  $p_h$  as  $p_h > r$ . Then  $\bar{F}(d_2(Q; p_h))$  is increasing in  $p_h$ . Since the single-crossing condition implies that  $p_h \leq p'_h$ , we have  $(p_h - w)\bar{F}(d_2(Q; p_h)) \leq (p'_h - w)\bar{F}(d_2(Q; p'_h))$ . Since the single spread preserves the mean, i.e.,  $\alpha p_h + (1 - \alpha)p_l = \alpha' p'_h + (1 - \alpha')p'_l$ , from the first-order condition (5), we can observe that

$$\frac{\partial \Pi_{G_p}(Q_{iF_p}^*; \lambda)}{\partial Q} \geq \frac{\partial \Pi_{F_p}(Q_{iF_p}^*; \lambda)}{\partial Q} = 0,$$

where  $\Pi_{G_p}$  and  $\Pi_{F_p}$  represent the expected utility function corresponding to  $G_p$  and  $F_p$ , respectively. Then, the concavity of  $\Pi_{G_p}$  implies that  $Q_{iG_p}^* \geq Q_{iF_p}^*$ .

A.7. Proof of Proposition 8

Under the two-wholesale-price contract  $(w, \bar{p})$ , the expected utility function can be expressed as

$$\tilde{\Pi}(Q; \lambda) = E\pi(Q; \xi, \bar{p}) + (\lambda-1) \left[ \int_0^{d_1(Q)} \pi_1(Q; \xi, \bar{p}) dF(\xi) + \int_{d_2(Q; \bar{p})}^\infty \pi_2(Q; \xi, \bar{p}) dF(\xi) \right]. \tag{1}$$

Similar to the single-wholesale-price contract, it is obvious that  $\tilde{\Pi}(Q; \lambda)$  is concave in  $Q$ , and there exists a unique optimal solution and its first-order condition is

$$(\bar{p}-w) - (\bar{p}-v)F(\tilde{Q}_\lambda^*) + (\lambda-1)[(\bar{p}-w)\bar{F}(d_2(\tilde{Q}_\lambda^*; \bar{p})) - (w-v)F(d_1(\tilde{Q}_\lambda^*))] = 0. \tag{2}$$

Inserting  $Q_\lambda^*$ , the optimal initial order quantity under the one-price only contract, into the left-hand side of (2), we know from (5) that if

$$\bar{p}\bar{F}(Q_\lambda^*) + (\lambda-1)(\bar{p}-w)\bar{F}(d_2(Q_\lambda^*; \bar{p})) \leq \bar{p}\bar{F}(Q_\lambda^*) + \alpha(\lambda-1)(p_h-w)\bar{F}(d_2(Q_\lambda^*; p_h)),$$

then  $\partial \tilde{\Pi}(Q_\lambda^*; \lambda) / \partial Q \leq \partial \Pi(Q_\lambda^*; \lambda) / \partial Q$ . By the concavity of the expected utility,  $\tilde{Q}_\lambda^* \leq Q_\lambda^*$ . Otherwise,  $\tilde{Q}_\lambda^* > Q_\lambda^*$ .

In particular, if  $\tilde{p} \leq \min(r, \bar{p})$ , then  $d_2(Q_i^*; \tilde{p}) = 0$ , which implies that

$$\begin{aligned} \frac{\partial \tilde{\Pi}(Q_i^*; \lambda)}{\partial Q} &= (\tilde{p} - w) - (\tilde{p} - v)F(Q_i^*) - (\lambda - 1)(w - v)F(d_1(Q_i^*)) \\ &\leq (\bar{p} - w) - (\bar{p} - v)F(Q_i^*) - (\lambda - 1)(w - v)F(d_1(Q_i^*)) \\ &\leq (\bar{p} - w) - (\bar{p} - v)F(Q_i^*) + (\lambda - 1)[(p_h - w)\bar{F}(d_2(Q_i^*; p_h)) \\ &\quad - (w - v)F(d_1(Q_i^*))] \\ &= \frac{\partial \Pi(Q_i^*; \lambda)}{\partial Q} = 0. \end{aligned}$$

By concavity,  $\tilde{Q}_i^* \leq Q_i^*$ .

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